Model Appendix for "How Start-up Firms Innovate: Technology Strategy, Commercialization Strategy, and their Relationship"

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This 'model appendix' provides a formal two-stage, complete information, economic model of the 'system vs. component' theory of innovation described in Egan (2013a), *How Start-up Firms Innovate: Technology Strategy, Commercialization Strategy, and their Relationship*. Egan (2013b), *Venture Capitalists as Vendors of Complementary Components*, embeds this theory in a heterogenous-cost Cournot oligopoly market structure, adds considerations of the allocation of surplus through bargaining strength, and provides an analysis of the welfare consequences of entry and acquisition. Moreover, Egan (2013b) frames the model in the context of the literature of incomplete contracting and discusses information asymmetries, moral hazard, and other aspects that are central to the relationships between entrepreneurs, incumbents, and financial intermediaries like venture capitalists. Comparative statics on the formal model can be used to derive the hypotheses in the main body of Egan (2013a) and provide foundations for the additional results included in its footnotes.

1 An Economic Model of the System vs. Components Theory of Innovation

1.1 Players and Primitives

A start-up firm, S, and a 'representative' incumbent, I, will play a two stage game with complete information.[1] I begin by assuming that production of a good requires a system made up of two industry-specific 'components'. Using two components allows a tractable analysis and provides the intuition for what happens with n components.

The normalized qualities of the components will be non-negative, and denoted $A : [0, \bar{A}]$ and $B : [0, \bar{B}]$, where $\bar{A}$ and $\bar{B}$ are positive real upper-bounds. One might imagine that in some industry the good consists of a visual display screen and audio speakers. The quality of the screen might be measured in pixels per inch (PPI) and the quality of the speakers might be measured in terms of total harmonic distortion (THD). A and B are normalized

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[1] Each incumbent sells its goods in an oligopolistic industry. The 'representative' incumbent is either any incumbent if all incumbents have the same system qualities or the incumbent that would have the highest willingness-to-pay (net of any costs) for the start-up firm's technological components if the incumbents' system qualities differ.

measures, giving values on the positive real number line, to enable comparison of qualities. When $A = B$, the normalized qualities of the two components will be equal.[2]

Both the start-up and the incumbent will produce a two-component-based good in the same sector of the economy. Public domain components, A_{pd} and B_{pd}, and the incumbent's patented components, A_I and B_I, are exogenously provided. The incumbent's patented components are assumed to be of strictly higher quality than the public domain components: $A_I > A_{pd}$, $B_I > B_{pd}$, and $\min\{A_I, B_I\} > \max\{A_{pd}, B_{pd}\}$. For ease of analysis, I will assume that the incumbent's technologies are of the same quality so that $A_I = B_I$.[3]

The quality of a system, $f : \mathbb{R}^{2+} \rightarrow \mathbb{R}^+$, of two components will be $f(A, B) = AB$. This multiplicative relationship between component qualities and system quality creates the complementarity between components.[4] Thus the incumbent's system quality is $A_I B_I$.

A firm that sells its good in the market will realize a firm value V, based on its system quality so that $V : \mathbb{R}^+ \rightarrow \mathbb{R}^+$.[5] I will assume that for the incumbent $V_I(f_I) = f_I = A_I B_I$. This assumption yields a neat algebraic solution when calculating the equilibria. The start-up firm will have an additional factor in its value function; when it undertakes an IPO it will pay a fee of ϕ. Underwriting expenses, regulatory costs, and other fees, make initial public offerings much more expensive than acquisitions for a start-up. Accordingly, ϕ will represent the costs of an IPO less the costs of an acquisition.

Before the game begins, nature provides the start-up firm with a technological opportunity $\tau : [A_{pd}+B_{pd}, \bar{A}+\bar{B}]$. A start-up firm is endowed with an R&D budget normalized to 1, that it can invest in producing its components, A_S and B_S. One should think of τ as a measuring the quality of components that the start-up firm can create, relative to both the public domain technologies and the incumbent's components.[6] The lower bound for τ

[2]In the screen and speakers example, this should not be taken to mean that PPI equals THD. Instead it will mean that relative to the PPI of other screens and relative to the THD of other speakers, the quality of the screen is equal to the quality of the speakers.

[3]This assumption is benign – differences in the qualities of the incumbent's components generally support specialization and acquisition as an equilibrium outcome over a greater range of parameters but do not change the qualitative implications of the model. In particular, when the incumbent's weakest technology component stands opposite the highest quality public domain component, the equilibrium of specialization and acquisition (described in the next section) becomes very easy to support. Small deviations in the incumbent's system quality always make the equilibrium described in this paper easier to support. Large deviations are only problematic when the best available public domain component is of the same type (i.e., A or B) as the incumbent's highest-quality component and there is a large difference in the quality of public domain components.

[4]A wide range of Cobb-Douglas production functions for system quality (i.e., $\gamma A^{\lambda} B^{\kappa}$) provide the same qualitative results. The results are not dependent on increasing returns to scale; system quality functions with constant and decreasingly returns to scale work as well. Using the system quality function $f(A, B) = AB$ greatly simplifies the analysis.

[5]Competitive effects are added to the model by assuming that firm value V is based both on firm's system quality and the quality of a new rival (if any), so that $V : (\mathbb{R}^+, \mathbb{R}^+) \rightarrow \mathbb{R}^+$. Denoting the firm of interest as i and its rival as j, the value of the firm will be increasing in f_i and decreasing in f_j, so that $\frac{\partial V}{\partial f_i} \geq 0$ and $\frac{\partial V}{\partial f_j} \leq 0$. I assume that $V_i(f_i, 0) < V_i(f_i, f_j) + V_j(f_j, f_i)$ to prevent mergers for market power reasons alone. The value function therefore abstracts away from the market structure while retaining the notion that there will be dissipation of rents due to competition when the start-up firm enters the market. Adding competitive effects alters the parameterization of the model without changing its substantive predictions and is discussed later.

[6]As τ is just a real number, there is a linear trade-off for the start-up firm between investing in both

is chosen so that a start-up firm can choose to best both public-domain components.[7] The upper bound for τ limits a start-up firm to creating the highest possible quality proprietary components.

1.2 Sequence, Actions and Payoffs

In the first stage of the game the start-up firm chooses a weight, $\omega \in [0, 1]$, to allocate its R&D budget between its two components. When a start-up firm produces a component of higher quality than is available from the public domain, the component is patented. When a start-up firm produces a component of lower quality than is available from the public domain, the component is not patented and the start-up firm uses the public domain component instead. The start-up firm's system quality following its choice of ω is f_S. If the start-up firm produced two proprietary components $f_S = A_S B_S$, otherwise $f_S = A_S B_{pd}$ or $f_S = A_{pd} B_S$.

In the second stage of the game, the incumbent decides whether or not to acquire the start-up firm. If the incumbent does not acquire the start-up firm, the start-up firm will pay a cost ϕ, undergo an initial public offering (IPO), and enter the market with a value $V_S = \max\{0, f_S - \phi\}$. I will assume that $\phi < A_I B_I$, so a start-up firm that achieves the same system quality as the incumbent can undergo an IPO. If the incumbent acquires the start-up firm, the combined firm, denoted IS, will use the best of each of the A and B components of the start-up firm and the incumbent, and the incumbent's component whenever there is a tie. Therefore the combined firm will have a system quality $f_{IS} = A_{IS} B_{IS}$ and value $V_{IS} = A_{IS} B_{IS}$.

The surplus, S, to an acquisition is the difference between the value of the combined firm, V_{IS}, and the value of the two firms as independent entities $(V_I + V_S)$:

$$S = V_{IS} - V_I - V_S \tag{1}$$

In an acquisition each player will receive their outside option (V_I or V_S) plus a share of the surplus. It is therefore a dominant strategy for the incumbent to choose an acquisition whenever the surplus is positive and to choose an IPO (for the start-up firm) whenever the surplus is not positive.

The share of the surplus that each receives will depend on their bargaining strength. The share accruing to the start-up firm will be denoted $\beta \in (0, 1)$ and the share accruing to the incumbent will be denoted $(1 - \beta)$.[8]

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components or a single component. An extension to the model would have nature provide the start-up firm with a technological opportunity function. This would allow the inclusion of increasing or decreasing returns to specialization.

[7] A lower bound of $\min\{A_{pd}, B_{pd}\}$ is sufficient for a start-up firm to best one public domain component and so create one proprietary component. This is necessary for a start-up firm to appear in my dataset. However, nothing interesting happens in the model until it is potentially optimal for a start-up firm to consider investing in the development of both components. This is proved in equation 4 in the appendix.

[8] If there are many incumbents and only one start-up firm we would expect that monopoly over the 'supply' of the start-up firm's technology will allow the start-up firm to secure all of the surplus (i.e., $\beta \to 1$). Alternatively, monopsony for the incumbent would suggest that $\beta \to 0$.

The payoffs to the start-up firm are therefore:

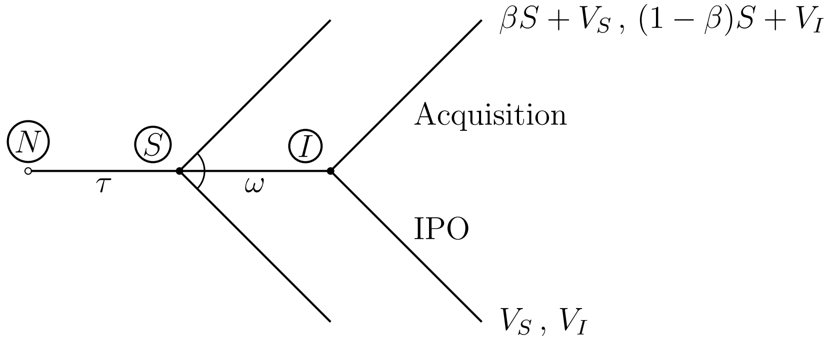
$$\pi_S = \begin{cases} \beta S + V_S = \beta\left(V_{IS} - V_I\right) + (1-\beta)V_S & \text{if } S > 0 \\ V_S & \text{if } S \leq 0 \end{cases} \tag{2}$$

And the payoffs to the incumbent are therefore:

$$\pi_I = \begin{cases} (1-\beta)S + V_I = (1-\beta)\left(V_{IS} - V_S\right) + \beta V_I & \text{if } S > 0 \\ V_I & \text{if } S \leq 0 \end{cases} \tag{3}$$

The game, including the draw of τ by nature, is shown in figure 1 below.

Figure 1: The game (including the draw of τ by nature)



1.3 Equilibrium Strategies

The start-up firm can maximize its system qualities in two ways. Either it relies on the public domain for one component and invests everything in development of the other component, or it spreads its R&D budget (and so its draw of τ) over both components. I will refer to these solutions as the corner and interior solutions, respectively.

The start-up firm's possible system qualities are:[9]

$$f_S = \begin{cases} A_{pd} \cdot \tau(1-\omega) \ \mid \ \omega \in [0, \underline{\omega}] & \text{if using } A_{pd} \\ \tau\omega \cdot \tau(1-\omega) \ \mid \ \omega \in [\underline{\omega}, \overline{\omega}] & \text{if both components are proprietary} \\ \tau\omega \cdot B_{pd} \ \mid \ \omega \in [\overline{\omega}, 1] & \text{if using } B_{pd} \end{cases} \tag{5}$$

[9]The bounds of the interior solution, $\underline{\omega}$ and $\overline{\omega}$ are derived in equation 4 below. When τ is less than $A_{pd} + B_{pd}$, no interior solution exists.

$$\begin{aligned} A_{pd} = \tau\omega \quad &\Longrightarrow \quad \underline{\omega} = \frac{A_{pd}}{\tau} \\ B_{pd} = \tau(1-\omega) \quad &\Longrightarrow \quad \overline{\omega} = \frac{\tau - B_{pd}}{\tau} \\ \underline{\omega} = \overline{\omega} \quad &\Longrightarrow \quad \tau = A_{pd} + B_{pd} \end{aligned} \tag{4}$$

When the start-up firm uses a public domain component, there are two possibilities. $A_{pd} \cdot \tau(1-\omega)$ achieves its maximum when $\omega = 0$. On the other hand, $\tau\omega \cdot B_{pd}$ achieves its maximum when $\omega = 1$. Without loss of generality, label the lower quality of the public domain components A_{pd}. The start-up firm then maximizes its corner system quality, $f_S^c = \tau B_{pd}$, when $\omega^c = 1$. This will be the specialization technology strategy.

The start-up firm can also potentially maximize its system quality by choosing the interior solution $\omega^* = \frac{1}{2}$, when this is feasible (i.e., $\underline{\omega} \leq \frac{1}{2} \leq \overline{\omega}$). This will be the generalization technology strategy. It is found by taking a first-order condition of the middle equation of equation 5 with respect to ω and gives a system quality of $f_S^* = \frac{\tau^2}{4}$.

The equilibrium actions of the model change according to the draw of τ (the technological opportunity provided by nature). The equilibrium of the model is solved by calculating four 'derived parameters', labeled $\hat{\tau}$, $\tilde{\tau}$, and $\underline{\tau}$ and $\overline{\tau}$, each expressing a threshold value of τ in terms of the exogenously given parameters A_{pd} and B_{pd} (the public domain component qualities), A_I and B_I (the incumbent's component qualities), β (the bargaining strength), and ϕ (the cost of an IPO). Whether τ is above or below each of these thresholds will, together, tell us the equilibrium actions that the start-up firm and the incumbent will take. Put another way, these derived parameters together determine the best-response functions of the both the start-up firm and the incumbent over all possible values of τ.

$\hat{\tau}$ considers when the start-up firm prefers to switch from specialization to generalization when there is no acquisition surplus and so the incumbent's best response is to choose an IPO for the start-up firm. As ϕ must be paid regardless, $\hat{\tau}$ is found by equating f_S^c and f_S^*: $\hat{\tau} = 4B_{pd}$.[10] The start-up firm cannot create acquisition surplus with a general strategy. A choice of ω^* equates the qualities of the start-up firm's components (i.e., $A_S = B_S$), so that the start-up firm's component's qualities never straddle the incumbent's qualities.[11] The start-up firm can, however, create acquisition surplus by choosing specialization providing that there is sufficient technological opportunity to best the incumbent's weakest component (i.e., $\tau > \min\{A_I, B_I\}$).

$\tilde{\tau}$ marks the point where acquisition surplus becomes available if the start-up firm chooses specialization. It is found by setting the surplus S, in equation 1, to zero.[12] Substituting

[10] As $\tau \geq A_{pd} + B_{pd}$, $\hat{\tau} = 0$ is not a possible solution.

[11] A start-up firm cannot create acquistion surplus with an incumbent if the qualities of its components do not straddle the qualities of the incumbent's components. From equation 1, and as by assumption $V_S \geq 0$ and $\phi < A_I B_I$:

$$\begin{aligned} \max\{A_S(\omega,\tau), B_S(\omega,\tau)\} \leq \min\{A_I, B_I\} &\implies (A_{IS} = A_I) \wedge (B_{IS} = B_I) \\ \therefore S = A_I B_I - A_I B_I - &\underbrace{V_S}_{\geq 0} \leq 0 \end{aligned} \tag{6}$$

$$\begin{aligned} \min\{A_S(\omega,\tau), B_S(\omega,\tau)\} > \max\{A_I, B_I\} &\implies (A_{IS} = A_S) \wedge (B_{IS} = B_S) \\ \therefore S = A_S B_S - A_I B_I - A_S B_S + &\underbrace{\phi}_{<A_I B_I} \leq 0 \end{aligned} \tag{7}$$

If the start-up firm chooses the internal optimum ω^* then $A_S = B_S$ and it must be the case (recalling that $A_I = B_I$) that either the condition in either equation 6 (both of the start-up firm's components are weakly inferior to both of the incumbent's components) or equation 7 (both of the start-up firm's components are superior to both of the incumbent's components) holds. Adding competitive effects to the model does not undermine this result as, by assumption, $V_i(f_i, 0) < V_i(f_i, f_j) + V_j(f_j, f_i)$.

[12] Equation 8, below, calculates $\tilde{\tau}$, the point at which acquisition surplus becomes available to a special-

$f_S^c - \phi$ for V_S and setting $V_{IS} = \tau B_I$, we find that $\tilde{\tau} = \frac{A_I B_I - \phi}{B_I - B_{pd}}$. Below $\tilde{\tau}$ there is no acquisition surplus and the incumbent's best response to specialization is to choose an IPO, whereas above $\tilde{\tau}$ there is acquisition surplus and the incumbent's best response to specialization is to choose an acquisition.

Just because acquisition surplus is available doesn't necessarily mean that the start-up firm would want to specialize. It may prefer to pursue a general strategy, create a rival system of components, and 'go it alone' instead. $\underline{\tau}$ and $\overline{\tau}$ provide the lower and upper bounds, respectively, of the range over which (if acquisition surplus is available) the start-up firm would prefer to specialize. Outside of this range, the start-up firm would prefer a general strategy. $\underline{\tau}$ and $\overline{\tau}$ are found by equating the difference between the start-up firm's payoff from specialization and an acquisition and the start-up firm's payoff from generalization and an IPO to zero. As this difference is a quadratic, it yields two roots. The equations for these roots are centered around $2\left(\beta B_I + (1-\beta)B_{pd}\right)$, and their separation is a function of ϕ.[13]

The equilibrium actions taken by the start-up firm and the incumbent depend both on the draw of τ and the ordering of the four derived parameters. When $\tilde{\tau} > \overline{\tau}$, there is never

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ization strategy:

$$\begin{aligned} S &= \overbrace{\tau B_I - A_I B_I}^{V_{IS}{}^c - V_I} - \overbrace{(\tau B_{pd} - \phi)}^{V_S{}^c} \\ \therefore S = 0 \implies \tilde{\tau} &= \frac{A_I B_I - \phi}{B_I - B_{pd}} \quad \text{and} \quad \frac{\partial S}{\partial \tau} = B_I - B_{pd} \end{aligned} \tag{8}$$

With $A_I = B_I$, $\tilde{\tau} = \frac{B_I^2 - \phi}{B_I - B_{pd}}$. As $A_I B_I > \phi$ and $B_I > A_{pd}$, we are reassured that $\tilde{\tau} > 0$ and $\frac{\partial S}{\partial \tau} > 0$. Thus as τ increases, it will reach $\tilde{\tau}$ and beyond this point an acquisition will have positive surplus.

[13] Equation 9, below, solves for the points $\underline{\tau}$ and $\overline{\tau}$ such that the difference, D, between the payoff to the start-up firm from an acquisition under ω^c and the payoff to the start-up firm from an IPO under ω^* is equal to zero. It also shows that between these values of τ the start-up firm prefers to specialize and so secure an acquisition, whereas above and below these values of τ, the start-up firm prefers a general strategy and so to secure an IPO.

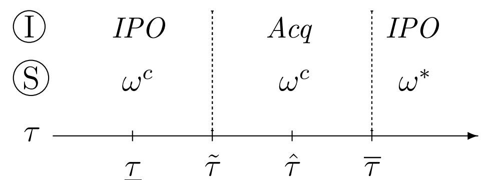
$$\begin{aligned} D &= \overbrace{\left(\beta(\tau\omega^c \cdot B_I - A_I B_I) + (1-\beta)(\tau\omega^c \cdot B_{pd} - \phi)\right)}^{\beta(V_{IS}{}^c - V_I) + (1-\beta)V_S{}^c} - \overbrace{\left(\tau\omega^* \cdot \tau(1-\omega^*) - \phi\right)}^{V_S{}^*} \\ \therefore D &= \tau\left(\beta B_I + (1-\beta)B_{pd}\right) - \frac{\tau^2}{4} + \beta(\phi - A_I B_I) \\ D &= 0 \implies \\ \underline{\tau} &= 2\left(\beta B_I + (1-\beta)B_{pd} - \sqrt{\left(\beta B_I + (1-\beta)B_{pd}\right)^2 + \beta(\phi - A_I B_I)}\right) \\ \overline{\tau} &= 2\left(\beta B_I + (1-\beta)B_{pd} + \sqrt{\left(\beta B_I + (1-\beta)B_{pd}\right)^2 + \beta(\phi - A_I B_I)}\right) \\ \frac{\partial D}{\partial \tau} &= \left(\beta B_I + (1-\beta)B_{pd}\right) - \frac{\tau}{2} \quad \therefore \left.\frac{\partial D}{\partial \tau}\right|_{\tau=\underline{\tau}} > 0 \quad \text{and} \quad \left.\frac{\partial D}{\partial \tau}\right|_{\tau=\overline{\tau}} < 0 \end{aligned} \tag{9}$$

Rearranging $\beta B_I + (1-\beta)B_{pd}$ gives $B_{pd} + \beta(B_I - B_{pd})$. As $B_I > B_{pd}$ by assumption, and as $\beta \in (0,1)$, this term is guaranteed non-negative. The inside of the square roots in $\underline{\tau}$ and $\overline{\tau}$ are non-negative, and so the square root term itself is real, for almost all parameter values. For example, when $\beta \to 1$, as $A_I = B_I$, $\beta^2 {B_I}^2 = \beta A_I B_I$ and the roots are non-negative. At the other extreme, when $\beta \to 0$, the inside of each root approaches ${B_{pd}}^2$, which is strictly positive by assumption. Moreover, $\underline{\tau}$ and $\overline{\tau}$ are symmetric about the point $\overline{\underline{\tau}} = 2B_{pd} + 2\beta(B_I - B_{pd})$, where the difference in value between choosing specialization (and securing an acquisition) and a general strategy (and securing an IPO) peaks.

any acquisition surplus in equilibrium and so the incumbent's best response is always to choose an IPO for the start-up firm. I will discard this uninteresting case. When $\tilde{\tau} \geq \underline{\tau}$ there are three equilibrium actions, consistent with the theory of system vs. components described in the main body of this paper.[14]

There is a consistent pattern to how the three equilibrium actions progress as the technological opportunity, τ, increases. For low values of τ the equilibrium actions are (ω^c, IPO); when τ is the 'intermediate' range $\tau \in [\max\{\tilde{\tau}, \underline{\tau}\}, \overline{\tau}]$ the equilibrium actions are (ω^c, Acq); and for high values of τ the equilibrium actions are (ω^*, IPO). Figure 2, below, shows the ordering of the derived parameters and the three equilibrium actions.

Figure 2: Patterns of actions within equilibrium strategies



1.4 The Effects of Competition

Competition in the same product market leads to rent dissipation. When the start-up firm enters the market through an initial public offering both the start-up firm and the incumbent will be worth less when competitive effects are greater. Acquisition surplus therefore increases as rent dissipation from competition increases – the merged firm gains additional value from the lack of competition. Under most commonly assumed oligopoly models, adding competitive effects back into the model has very predictable consequences.[15]

$\hat{\tau}$ is unchanged by the inclusion of competitive effects because $\hat{\tau}$ is conditional on entry where competition is unavoidable. Increased competition will decrease $\tilde{\tau}$ (the threshold

[14]When $\tilde{\tau} < \underline{\tau}$, there are four equilibrium actions to this model. The fourth equilibrium action is (ω^*, IPO). This action occurs in a region between the low values of τ and the bottom of the intermediate range (i.e., $\max\{\tilde{\tau}, \underline{\tau}\}$). An comparison of failed venture-capital-backed firms againsts all successful start-up firms was consistent with the existence of a four action equilibrium. Comparative statistics on $\tilde{\tau} - \underline{\tau}$ suggest that a three action equilibrium is more likely than a four action equilibrium when: the rent dissipation due to new entry is large; the public domain components (A_{pd} and B_{pd}) are close in quality to the incumbent's components (A_I and B_I), or equivalently when a dominant design has not yet been established in an industry and so patented components are less valuable; and the cost of an IPO beyond the cost of an acquisition (ϕ) is high. Firms that secure venture capital investment are more likely to operate in nascent sectors where the rent dissipation due to new entry is high and where dominant designs may not yet have been established.

[15]The definition of surplus, S (originally stated in equation 1), including competition effects can be summarized as:

$$S = V_{IS}\left(f_{IS}(\tau), 0\right) - V_I\left(f_I, f_S(\tau)\right) - V_S\left(f_S(\tau), f_I\right) \tag{10}$$

The first term in equation 10 has '0' as a second parameter because there is no new competition to affect rents when the incumbent acquires the start-up firm. The second and third terms concern what happens to the value of the incumbent and value of the start-up firm when the start-up firm enters the market through an IPO. Both of these values are reduced by competitive effects, so surplus is increased when the rent dissipation due to competition increases. The difference, D (previous stated in equation 9), between the payoffs (see equation 2) when surplus is available to (ω^c, Acq) and (ω^*, IPO), can be re-expressed to include at which surplus is positive): because there are now gains to be made from merging to reduce competitive effects, firms will merge with lower gains from complementarities. And when competition increases, $\underline{\tau}$ and $\overline{\tau}$ separate, with $\underline{\tau}$ decreasing and $\overline{\tau}$ increasing, as the returns to market power supplement the complementarities between the start-up firm and the incumbent. Taken together, the competition related changes to derived parameters have three effects: acquisitions taken place at lower values of technological opportunity; acquisitions are supported over a greater range of realizations of technological opportunity; and the equilibrium is relatively more likely to consist of three than four actions. This last point is a consequence of $\hat{\tau}$ remaining fixed while $\tilde{\tau}$ moves down – $\tilde{\tau}$ is less than or equal to $\hat{\tau}$ in each three equilibrium action ordering of derived parameters. Nevertheless, start-up firms and incumbents still pursue the same equilibrium strategies as before, just with different threshold parameters.

1.5 Comparative Statics for the Effect of Sarbanes-Oxley

The introduction of the 2002 Sarbanes-Oxley act raised the cost of an IPO but did not directly affect the cost of an acquisition. As a result ϕ increased. In the model this has predictable consequences on $\hat{\tau}$, $\tilde{\tau}$, and $\underline{\tau}$ and $\overline{\tau}$:

$$\begin{aligned}
&\frac{\partial \hat{\tau}}{\partial \phi} = 0 \\
&\frac{\partial \tilde{\tau}}{\partial \phi} = -\frac{1}{B_I - A_{pd}} < 0 \\
&\frac{\partial \underline{\tau}}{\partial \phi} = -1 \cdot \Big(\big(\beta B_I + (1-\beta) B_{pd}\big)^2 + \beta(\phi - A_I B_I) \Big)^{-\frac{1}{2}} \cdot \beta < 0 \\
&\frac{\partial \overline{\tau}}{\partial \phi} = +1 \cdot \Big(\big(\beta B_I + (1-\beta) B_{pd}\big)^2 + \beta(\phi - A_I B_I) \Big)^{-\frac{1}{2}} \cdot \beta > 0
\end{aligned} \tag{12}$$

competition effects as:

$$D = \beta\Big(V_{IS}\big(f_{IS}^c(\tau), 0\big) - V_I\big(f_I, f_S^c(\tau)\big)\Big) + (1-\beta)\, V_S\big(f_S^c(\tau), f_I\big) - V_S\big(f_S^*(\tau), f_I\big) \tag{11}$$

Like surplus, this difference also increases with competition. As a result, when surplus is available, the equilibrium actions (ω^c, Acq) are more easily supported. Closed-form solutions for the derived parameters of the model ($\hat{\tau}$, $\tilde{\tau}$, $\underline{\tau}$, and $\overline{\tau}$) are only possible with explicit assumptions concerning V. For example, one could model firm values by assuming heterogeneous-cost Cournot competition (with $N > 1$ incumbents) as a market structure.