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Patent Informatics for Patent Thicket Detection:
A Network Analytic Approach for Measuring the Density of Patent Space

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Abstract

When organizations in technology industries attempt to advance their innovative activities, they may encounter patent thickets, or dense webs of overlapping intellectual property rights owned by different companies that must be hacked through in order to commercialize new technology. Throughout the last 150 years, however, organizations have stumbled into a number of patent thickets and have occasionally responded by constructing patent pools or organizational structures where multiple firms collectively aggregate patent rights into a package for licensing, either among themselves or to any potential licensees irrespective of membership in the pool. Such collaboration among technologically competing firms, however, has often encountered difficulty from an antitrust standpoint, even if the formation of the pool is pro-competitive.

Despite all that has been written lamenting the problem of patent thickets, the antitrust regime has never had an objective method of verifying the existence of a patent thicket in a given section of patent space. In response to the lack of such a methodology, this paper proposes a tool to facilitate objectively demonstrating the existence of patent thickets.

This paper proposes a thicket identification methodology that uses a network analytic technique to determine if a patent pool is coincident with a patent thicket by comparing the density of the patent pool to the density of the surrounding patent space. This paper then applies the new methodology to two existing patent pools and verifies the existence of underlying patent thickets.

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I. Introduction

When organizations in technology industries attempt to advance their innovative activities, they may encounter patent thickets, or dense webs of overlapping intellectual property rights owned by different companies that must be hacked through in order to commercialize new technology (Shapiro 2000). Despite all that has been written lamenting the problem of patent thickets,[1] an objective methodology for verifying the existence of a patent thicket has never been developed. The situation is somewhat analogous to the astrophysical discussion of black holes. Astronomers developed a theoretical construct to describe a black hole long before the technology existed to verify the existence of such a phenomenon. While mankind has yet to physically encounter a black hole in space throughout the last 150 years, organizations have stumbled into a number of patent thickets and have occasionally responded by constructing patent pools or organizational structures where multiple firms collectively aggregate patent rights into a package for licensing, either among themselves or to any potential licensees irrespective of membership in the pool (Clarkson 2004b). Such collaboration among technologically competing firms, however, has often encountered difficulty from an antitrust standpoint, even if the formation of the pool is pro-competitive.

Just as the existence of black holes is a necessary but insufficient element in proving certain cosmological theories,[2] the existence of a patent thicket is a necessary but insufficient condition for demonstrating that a given collection of patents is a pro-competitive solution to a particular patent thicket problem (Clarkson 2004b). Unlike astrophysics researchers, however, those interested in solving patent thicket problems do not have a tool analogous to a radio telescope that can be used to verify the existence of a patent thicket in a given section of patent

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[1] Part II of this paper contains a review of that literature.

[2] *See, e.g.* "Black Holes and Beyond," http://archive.ncsa.uiuc.edu/Cyberia/NumRel/BlackHoles.html

space. In response to such a need, this paper proposes a new tool to facilitate objectively demonstrating the existence of patent thickets.

Using Shapiro's definition of a patent thicket as the starting point, two conditions must be satisfied in order for a collection of patents to be a patent thicket: the collection of patents must be both "dense" and "overlapping"(2000, pg. 120). By examining existing patent pools, this paper will demonstrate the identification and measurement of the first condition and will discuss theoretical possibilities for identification and measurement of the second.

Shapiro also provides a theoretical basis for establishing the existence of a patent thicket coincident with a patent pool. Building upon his premise that a patent pool is a natural market-clearing mechanism that forms within a patent thicket, it should be possible to verify that the density of patents within known pools is higher than the surrounding patent space. If the density measures of established pools are significantly higher than the density of their surrounding patent space, that finding will contribute a new dimension to the definition of patent thickets.

Part II of this article reviews the literature and prior research on patent thickets and patent pools, and Part III further discusses the two major questions that should be asked when examining a patent pool that is purportedly attempting to solve a patent thicket problem. Part IV proposes a thicket identification methodology that uses a network analytic technique to determine if a patent pool is coincident with a patent thicket. In order to validate the utility of the proposed methodology, Part V introduces two separate patent pools and then applies the proposed methodology to verify the existence of underlying patent thickets. Part VI concludes the paper with a discussion of the contributions and limitations of the proposed methodology for patent thicket identification.

II. Background

A. The Problem of Patent Thickets

Patent thickets are not a new phenomenon, and when the total number of owners of the conflicting intellectual property rights is small, the response to the patent thicket problem has often been to cross-license (Grindley & Teece 1997; Teece 1998; Teece 2000). When more than two parties are involved, however, the transaction costs of cross-licensing between all of the parties can be prohibitive, and additional economic barriers exist such as hold-ups and double marginalization (Viscusi *et al.* 2000). In response to these challenges throughout the last 150 years, organizations have attempted to solve the multi-party patent thicketing problem by constructing patent pools. Usually, each firm assigns or licenses its individual intellectual property rights to a specific entity that in turn exploits the collective rights by licensing, manufacturing, or both. Different licensing arrangements are then available, depending on whether the licensee is a member of the pool and how the resulting royalties are subsequently distributed among the members of the pool.

B. Prior Research on Patent Thickets and Patent Pools

The problem of patent thickets has recently caught the attention of much of the scientific and engineering community in a number of technological arenas (Clark *et al.* 2000; FTC 2002; FTC 2003; Glover 2002; Heller & Eisenberg 1998; Horn 2003; Lerner *et al.* 2003; Merges 1999; Newberg 2000). For example, firms in the semiconductor industry "find it all too easy to unintentionally infringe on a patent in designing a microprocessor, potentially exposing themselves to billions of dollars of liability and/or an injunction forcing them to cease production" (Shapiro 2000, p. 121). Heller and Eisenberg lament the "anticommons" in biomedical research due to the problem of patent thicketing (1998). Particularly in the biopharmaceutical industry, patent thickets threaten the process of cumulative innovation because they act "as barriers to entry [that prevent new entrants] from using the technologies protected by such patent thickets" (Glover 2002, p. C10).

A recent FTC report notes that in certain industries the large number of issued patents makes it virtually impossible to search all the potentially relevant patents, review the claims contained in each of those patents, and evaluate the infringement risk or the need for a license (FTC 2003). For the software industry the report cites testimony about the hold-up problems and points out "that the owner of any one of the multitude of patented technologies constituting a software program can hold up production of innovative new software" (2003, ch2, p. 3). For many firms, the only practical response to this problem of unintentional and sometimes unavoidable patent infringement is to file hundreds of patents each year so as to have something to trade during cross-licensing negotiations. In other words, the only rational response to the large number of patents in a given field may be to contribute to it.

Patent pools are perhaps an alternative response, but although the revenues generated from sales of devices based in whole or in part on patent pool technologies are at least $100 billion US per year (Clarkson 2003), the patent pooling phenomenon has received few scholarly treatments, and most of those have been historical in nature (Bittlingmayer 1988; Cassady 1959; Thomson 1987; Vaughan 1925). While some legal scholars have written favorably about patent pool formation (Merges 1996; Merges 1999; Newberg 2000), others have focused on potential competitive problems posed by patent pools (Carlson 1999; Priest 1977; Taylor 1992).

Economists have also examined patent pools, with some focusing on pools in the context of patent litigation settlements constrained by antitrust law (Choi 2003; Shapiro 2003), while others have examined pool formation generally (Gilbert 2002; Lerner *et al.* 2003; Lerner & Tirole 2002).

III. Patent Thicket Questions

In any analysis of a proposed solution to a purported patent thicket problem, two primary questions must be addressed. The first question is "Does a given collection of patents constitute a thicket?" The answer to this question is critical because the actual existence of a patent thicket is a necessary but insufficient condition for a pro-competitive combination. The second question is "What is the nature of the relationships between the patents in the thicket?" The standard taxonomy categorizes the economic relationship between individual patents as blocking, complementary, independent, or substitute (Andewelt 1984; Clarkson 2004b; Newberg 2000), or "BCIS." The elimination of substitutes is also an antitrust requirement for a pro-competitive pooling solution to a patent thicket (Priest 1977; USDOJ/FTC 1995).

Recent work by Clarkson (2004a) demonstrated the judicial importance of exploring these thicket questions when considering the legality of patent pooling arrangements, given that the antitrust and intellectual property regimes were frequently in tension for most of the 20th century, with patent pooling often facing rather aggressive antitrust enforcement even in situations where the pool was pro-competitive. Clarkson's study examined 101 cases of patent pool litigation between 1900 and 1970 and demonstrated that although the patent thickets underlying the respective patent pools were infrequently examined, their examination was potentially quite important for pool survival. In terms of overall litigation outcomes between 1900 and 1970, 21% of the identified patent pools survived litigation. When patent thickets were examined, however, 59% of those pools survived, and the study found a definite association between pool survival and the examination of thicket questions.

Although there is a clear temptation to immediately address the BCIS categorizations, without first developing a methodology for objectively identifying the existence of thickets, any attempts to segment thickets into BCIS categories would likely be problematic. Thus, although the next two sections will discuss both questions, this paper will only demonstrate a methodology for answering the first question. Answering the question of thicket existence, however, is a first step toward empirical usefulness of the theoretical BCIS framework.

A. *Existence of a Patent Thicket*

Although most of the recent analyses of patent pools have been economic, the economists have not provided a method for objectively determining the existence of a patent thicket. The nature of patents suggests, however, that a network analytic approach might prove instructive in identifying patent thickets. Social network analysis is a methodology developed by sociologists and organizational theorists to examine the social structure of groups. In this type of analysis, individuals are identified as the actors in a network, and the relationships between those actors are identified as ties. If the relationship from actor A to actor B can be different than that from actor B to actor A, the network is referred to as a directed network (or directed graph).

While social network analysis, as a science, has been most commonly applied to describe complex dynamics in human interaction, the underlying theory and methodology is not limited to interpersonal relationships. Network analysis describes the relationships among nodes, be they people, computers, power stations, or academic papers, as some form of resource that moves from one node to another. Network analysis has been applied to describe numerous human interaction webs – opinions and rumors (Watts 1999), diseases and epidemics (Newman 2002), and even terrorist cells (Carley et al. 2003; Krebs 2002). The dynamics of resources moving from one location to another is not specifically the province of human interaction. Network analysis has also been applied to uncover the nature of non-human phenomena, such as the power blackout in the eastern United States during the summer of 2003 or the spread of computer viruses (Newman et al. 2002).

Existing network analytic research in other areas of information sciences has concentrated on patterns of citation in literature and research (Price 1965; Price 1976; Redner 1998). Physicist Mark Newman has written extensively on the analysis of co-authorship networks within academic communities and scholarly publishing (2000; 2001). Patents share many similar characteristics – citation practices in particular – to academic works, and that research is quite relevant. Patent space as an information network bears significant similarity to academic citation networks on the basis of temporal limitations that specifically affect the directionality of linking vectors within a network. Similar to academic papers, a new entrant can only give citation to previous research, or "prior art." Because of this linear path, patents that give rise to increased innovation can be seen as significant in creating lineages or families of technologies – possibly the seeds from which a patent thicket grows (Freeman 1979).[3]

Previous work has demonstrated the methodological validity of using network analysis on patents. In an early study of patent networks, Podolny and Stuart (1995) developed the concept of a "technological niche" that included a focal innovation, the innovations on which the focal innovations built, the innovations that built upon the focal innovation, and the technological ties among the innovations within the niche. Using patents as the network nodes and patent citations as the network ties between nodes, they then were able to measure characteristics of innovation niches within the semiconductor industry to determine how subsequent innovations may or may not build upon the focal innovation. Those same authors used similar techniques in two subsequent articles. One article examined the evolution of

[3] As an example, the ISI Web of Knowledge (a scholarly citation database) indicates that Shapiro's patent thicket article has been cited at least 21 times in social science journals (4 times as a working paper and 17 times as the published article). Additionally, a quick Westlaw search reveals that the article has also been cited by at least 30 legal journals.

technological positions among firms (Stuart & Podolny 1996), and the other examined organizational survival within technological niches (Podolny et al. 1996).

Not only did these early studies establish the methodological validity of applying network analytic techniques to patent networks in general but also much of their analysis of technological niches and competitive crowding was based on a variation of network density, a fundamental network analytic concept (Marsden 1990; Wasserman & Faust 1994). To facilitate the identification of patent thickets within a larger patent space, the next section develops a new measure of patent thicket density.

IV. Objective Thicket Identification

As more and more patents issue, patent thickets become both denser and more numerous. Given that patent pools may be the only viable solution in certain instances, an objective methodology for demonstrating the existence of an underlying patent thicket would allow organizations attempting to form a patent pool to satisfy a necessary condition for the pool to be pro-competitive.

A. Exploring Patent Networks

To evaluate this threshold question of the existence or non-existence of a patent thicket, I propose a measure of patent thicket density. The standard network density equation (Wasserman & Faust 1994) for a directed network with g nodes

$$\Delta = \frac{\sum_{i=1}^{g}\sum_{j=1}^{g} x_{ij}}{g(g-1)} \qquad (1)$$

essentially counts up the total number of ties in a network and divides that total by the number of possible ties, where x_{ij} is the value of the tie from node i to node j. A core assumption of the standard density calculation is that each node in the network has a possible tie to each of the other nodes, an assumption which does not hold true for patents. In order to derive a density calculation for patent networks, it is necessary to deconstruct the standard calculation and then rebuild a patent-capable density calculation.

For a g-node network, each node n can cite g-1 other nodes. Thus the total possible number of nodes is $g(g-1)$, which is the denominator in the standard calculation. Individually, each node n has a local network density Δ_n, which equals the number of ties to and from node n divided by the total possible ties for that node, g-1.

$$\Delta_n = \sum_{j=1}^{g} \frac{x_{nj}}{g-1} \qquad (2)$$

Summing the local densities for all g nodes and dividing by g results in the standard density equation (1) above. Note that each local density has the same denominator g-1, which is only true if each node n can tie to each of the g-1 other nodes in the network.

That tying assumption does not hold true for patents, as any given patent can only cite patents that were issued previously. Subsequent patents cannot be cited by a prior patent, and thus the standard density equation cannot accurately correspond to patent network density.

B. Deriving Patent Network Density

Assuming a patent network with g patents, each node n can cite n-1 other patents.[4] Traversing the patent network chronologically, younger patents have more and more possible citations that they can make. The oldest patent in the network, however, will have zero possible citations to make,[5] which would result in an undefined local density for that patent. The local

[4] The network analytic term for citations made to other patents is "outdegrees." Citations received from other patents are called "indegrees."

[5] Citations to patents outside the network are discarded for the purposes of density calculations, as those prior patents do not constitute nodes in the network.

density for the oldest patent is thus discarded to avoid an undefined result.[6] Local patent density Δ_{np} for each subsequent patent n is derived by totaling up the citations actually made, or outdegrees, and dividing by the possible citations that could be made by that patent.

$$\Delta_{np} = \sum_{j=1}^{g} \frac{x_{nj}}{n-1} \qquad (3)$$

The average density for a patent network based on citations made is then derived by summing the remaining patent densities and dividing by g-1. Thus patent network density $\Delta_{p\text{-}out}$ is

$$\Delta_{p-out} = \frac{\sum_{n=2}^{g} \sum_{j=1}^{g} \frac{x_{nj}}{n-1}}{g-1} \qquad (4)$$

While Formula (4) is a density measure based on citations made, or outdegrees, it may be useful in certain circumstances to calculate patent network density using citations received, or indegrees.[7] Rewriting Formula (4) to use citations received requires a few modifications. Instead of discarding the local density of the oldest patent, the local indegree density for the youngest patent is discarded, as no other patents in the network can cite to it.

$$\Delta_{p-in} = \frac{\sum_{j=1}^{g} \sum_{n=2}^{g} \frac{x_{jn}}{n-1}}{g-1} \qquad (5)$$

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[6] A secondary reason to discard the local density for the oldest patent is related to the calculation of standard deviation. In a complete network, where all possible ties are in fact actual ties, average density is one and standard deviation is zero. Formula (4) yields an average patent density of one when the oldest patent is excluded (along with its undefined local density). The standard deviation of the local patent densities is also zero once the oldest patent is excluded.

[7] Citations received might be an indication of importance or knowledge flows (Jaffe & Trajtenberg 1999; Jaffe & Trajtenberg 2002; Jaffe *et al.* 1993), although the nature of patent citations casts significant doubt as to whether patent citations can be used as a direct proxy for knowledge transfer.

C. Applying Patent Network Density

After deriving a measure for patent network density, the next step is to apply it to a network and compare the results to the standard density calculation. Assume a six patent network as shown in Figure 1.

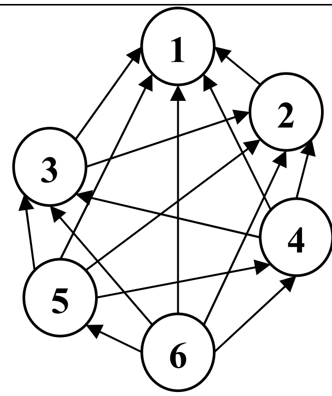

Figure 1. Patent Network A

If all possible citations are in fact actual citations, the resulting patent network is complete, which should result in an average density of one and a standard deviation of zero (Wasserman & Faust 1994). Formula (4) returns the appropriate result, as shown in Table 1. The indegree variant, Formula (5), also returns the appropriate result, as shown in Table 2.

Patent	Outdegree Citations	Possible Outdegree Citations	Local Density
1			
2	1	1	1.00
3	2	2	1.00
4	3	3	1.00
5	4	4	1.00
6	5	5	1.00
Average Density			1.00
Standard Deviation			0.00

Table 1. Patent Network Density for Network A (Citations Made/Outdegree).

Patent	Indegree Citations	Possible Indegree Citations	Local Density
1	5	5	1.00
2	4	4	1.00
3	3	3	1.00
4	2	2	1.00
5	1	1	1.00
6			1.00
Average Density			1.00
Standard Deviation			0.00

Table 2. Patent Network Density for Network A (Citations Received/Indegree).

Formula (1), the standard network density calculation, produces an entirely different result for Network A. Although Network A is a complete network from a patent citation standpoint, Formula (1) returns a density of 0.5.

Having established that "complete" networks are calculated appropriately using both measures of density, the next task is to examine networks that are less than complete. Assume a different six-patent network as shown in Figure 2, where the oldest patent is cited by each of the other patents, but no other citations are present in the patent network.

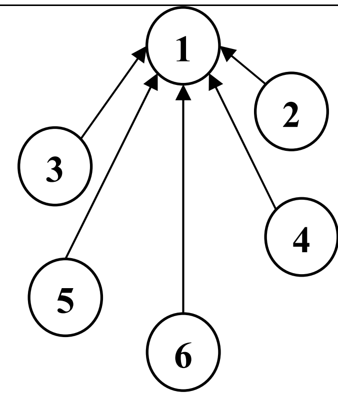

Figure 2. Patent Network B

In this scenario, the results returned by Formulas (4) and (5) are not the same.

Patent	Outdegree Citations	Possible Outdegree Citations	Local Density
1			
2	1	1	1.00
3	1	2	0.50
4	1	3	0.33
5	1	4	0.25
6	1	5	0.20
Average Density			0.46
Standard Deviation			0.32

Table 3. Patent Network Density for Network B (Citations Made/Outdegree).

Patent	Indegree Citations	Possible Indegree Citations	Local Density
1	5	5	1.00
2	0	4	0.00
3	0	3	0.00
4	0	2	0.00
5	0	1	0.00
6			
Average Density			0.20
Standard Deviation			0.45

Table 4. Patent Network Density for Network B (Citations Received/Indegree).

Both formulas, however, return higher average densities than Formula (1), which returns an average density of 0.1667.

Intra-network citations made by earlier patents are given more weight in Formula (4) because the denominator for each local density is the number of possible citations that can be made. Formula (5) behaves similarly, although citations made to more recent patents are given more weight. While treating such earlier or later citations as more important might seem appropriate for analyzing patent thickets and patent pools, a weighted density measure would likely be somewhat more robust.

Weighting each local density by the possible number of citations results in a weighted average patent network density Δ_p.

$$\Delta_p = \frac{\sum_{n=1}^{g}\sum_{j=1}^{g} x_{nj}}{g(g-1)/2} \qquad (6)$$

This formulation of patent network density has a number of advantages. Formula (6) still produces the proper result for a complete network and is simpler to calculate than either Formula (4) or (5). Additionally, as with Formula (1), calculating density based on citations made results in the same density for citations received.[8] Whereas the result of Formulas (4) and (5) will vary depending on which individual patents cite other patents like Formula (1), Formula (6) is not affected by variations in citation placement so long as the total number of citations remains the same.

D. Identifying the Existence of a Patent Thicket

In order to validate the measure of patent network density Δ_p, it would be useful to examine an area of the intellectual property space that is likely to have variation in densities. As discussed earlier, a logical starting point is Shapiro's suggestion that patent pools form where

[8] Note that this result might not be true in other types of networks where circular citations might be possible. Academic citations could present such a situation if two papers each cite the other. Such a citation pattern is impossible, however, in acyclic networks such as patent networks, and thus is not of concern here.

patent thickets already exist. If a patent pool is coincident with a patent thicket, then the density of the pool should be higher than the surrounding patent universe. As an alternative to calculating the density for the complete universe of patents in a given set of technology classes, a relevant near universe may be able to be constructed which should still provide a sufficient density contrast to identify a patent thicket. Although there has been relatively little empirical examination of network density (Marsden 1990; Wasserman & Faust 1994), both of these propositions can be stated as testable hypotheses:

H1: Patent network density Δ_p will be higher for a patent thicket than for the surrounding patent universe.

H2: Patent network density Δ_p will be higher for a patent thicket than for a relevant near universe.

In addition to the astrophysical analogy of using a radio telescope to identify black holes in interstellar space, this exercise could also be analogized to looking at a map of the United States that only displays roads and highways (i.e. no cities) and trying to identify where the cities are located based on the relative density of the roads.

1. NBER Patent Citation Data

Testing these hypotheses requires examining actual patent data. Previous work by Hall, Jaffe, and Trajtenberg (Hall et al. 2001) collected detailed information on almost 3 million U.S. patents granted between January 1963 and December 1999 and all citations made to these patents between 1975 and 1999 (over 16 million). This database was then made available by the National Bureau of Economic Research ("NBER"). The proposed methodology requires the use of two of the NBER patent data sets.

- A complete list of patents with designations of category, subcategory, and *n* class. These patents are contained in the SAS datafile pat63_99.tpt, available at http://www.nber.org/patents.

- A complete list of patent citations. These patents are contained in the SAS datafile cite75_99.tpt, available at http://www.nber.org/patents.

Within the NBER patent citation database, each patent is given two dimensions. The first value is the patent number, referred to as the *citing* value. The second value is the number of another patent to which the value connects, called the *cited* value. As is the nature of citation networks, the data are directional, always pointing from *citing* to *cited*.

2. Comparison of Network Densities

Once each subset of patents has been identified (i.e. pool, near universe, complete universe) and extracted from the NBER patent database, the particular subset of citing and cited pairs are extracted from the NBER patent citation database for each subset of patents, and the citing patents are numbered in ascending order. Once ordered, a count is taken of the number of cited entries per citing patent, $n(cited)$. Each patent in the list is then given a number, k, which represents the number of possible citations within the subset that are possible for the given patent. If n is the index number of the patent within the list, then $k=n-1$. The local unweighted density Δ_{np} is then calculated by dividing the number of actual citations by the number of citations possible for that patent, or $\Delta_{np}=n(cited)/k$.

After finding the local unweighted density, the weighted local densities of each patent are then calculated by multiplying the local unweighted density and the respective k value as a weight. The resulting list is used to calculate the standard deviation of the weighted densities using the k value as analytic weights. Using Formula (6), the weighted patent network density Δ_p is found by dividing the number of existing citations, $\sum n(cited)$, by the number of possible citations within the directed network graph, $g(g-1)/2$.[9] The same process is then repeated to

[9] In practice, both the average density and the standard deviation for Formula (6) can be calculated using a statistical package that can use analytic weights, such as STATA Thanks to Bill Simpson, Senior Statistician at the HBS Faculty Research Computing Center for help on developing the appropriate analytic weights. In this instance, the

generate a density measure for the surrounding patent space, and those two densities are then compared.[10] Just as a statistically significant difference in gravimetric readings from a radio telescope provides evidence of the existence of a black hole in interstellar space, a statistically significant difference in density provides evidence of a patent thicket within a larger area of patent space.

3. Shadow Pools

To assist in the validation of the network density measure Δ_p, the density of a given patent pool could also be compared to comparable "shadow pools," or collections of patents that match the pool on a number of dimensions. Formally, the development of the proposed shadow pools is accomplished in the following manner. Given a set of patents, A, numbered sequentially, a patent pool typically is a list of non-adjacent patents, B, drawn as a proper subset of the larger corpus.

$$B \subset A$$

For each pool patent *x* occupying a position in the set at position *i*, the corresponding patent in the *n*th shadow pool, B_n is the set of patents in A that is *n* positions away from *x*.

$$B_n = \{x_{i+n} \in A, \forall i \mid x_i \in B_0\}$$

If set A is bounded by date, for example, the number of patents within the shadow pool will decrease as $|n|$ grows larger and deviates further from the original positions.

As an example of how a shadow pool is constructed, assume a simplified patent universe of six patents (221, 287, 357, 481, 518, 533, and 612) and a small pool B_0 of three patents (357,

analytic weight for each observation of local patent network density is the number of possible citations for the given patent.

[10] Since each patent network density value Δ_p is the average of the individual local patent densities Δ_{np}, a *t*-test is used to compare the two mean values. The results of the *t*-test indicate the probability of a random collection of patents drawn from the given patent space having a density measurement as high as or higher than the density observed in the pool.

481, and 533). The shadow pools drawn from the six-patent universe would be constructed as follows:[11]

B_{-3}	B_{-2}	B_{-1}	B_0	B_1	B_2	B_3
221	221	221	221	221	221	221
287	287	287	287	287	287	287
357	357	357	357	357	357	357
481	481	481	481	481	481	481
518	518	518	518	518	518	518
533	533	533	533	533	533	533
612	612	612	612	612	612	612

Densities for these shadow pools can be compared against both the densities of the base pool B_0 as well as the density of the surrounding patent space A.

In order to demonstrate the utility of such an objective methodology for patent thicket identification, the next section introduces two recent patent pools and incorporates calculations of patent thicket density in an examination of their widely divergent fates.

V. Examining Patent Pools

The 1995 *Guidelines for the Licensing of Intellectual Property* ("*IP Guidelines*"), jointly issued by the U.S. Department of Justice ("DOJ") and the Federal Trade Commission ("FTC"), formally acknowledged that collective ownership structures for intellectual assets, including patent pools, could potentially be pro-competitive solutions to the patent thicket problem. While the *IP Guidelines* represent a welcome change in attitude by the antitrust enforcement regime, notably absent are any specific methodologies for examining a patent pool in the antitrust context. The lack of an objective methodology for evaluating patent pools soon became a significant factor in the destruction or survival of subsequently formed or examined patent pools.

[11] Note that pool B_{-3} contains only two patents each because any prior patents exist in a time before the earliest patent in the base pool B_0. Similarly pool B_3 contains only two patents each because any later patents exist in a time after the most recent patent in the base pool B_0.

A. Sample Pools

On June 26, 1997, the DOJ issued a Business Review letter indicating that a patent pool based on MPEG-2, a technology standard for compactly representing digital video and audio signals for consumer distribution, was not in violation of the antitrust laws of the United States. Less than a year later, however, on March 24, 1998, the FTC filed a complaint against a patent pool formed around photorefractive keratectomy ("PRK"), or laser eye surgery technology, and ultimately forced that pool to dissolve. One of the FTC litigators would later write that the PRK pool might actually have been a pro-competitive solution to a patent thicket (Newberg 2000), but by the time his article was published, the damage was done. If both the MPEG pool and the PRK pool were formed in response to the patent thicket problem, why did the antitrust regime destroy one pool and allow the other pool to live?

One significant difference between these two pools was that the MPEG pool was based on a third-party technical standard (Clarkson 2004b). That standard served as an alternate method of determining the existence of a patent thicket, as the standards document itself allowed for an objective assessment of essentialness of the patents in the pool. No such technological standard existed for PRK, however, and the lack of an objective methodology for assessing the existence of an underlying patent thicket proved fatal to the PRK pool (Clarkson 2004b). Had the methodology proposed in this paper been available to the FTC, would they have found evidence of an underlying patent thicket, given that the PRK pool was ultimately vindicated as pro-competitive? A comparison of results for both the MPEG and PRK pools suggests they would have.

B. Analytic Comparison

This section applies the methodology presented in Part IV to both the MPEG and PRK patent pools to see if they are coincident with underlying patent thickets.

1. Analyzing the MPEG Pool

The US patent numbers in the MPEG pool were downloaded from the MPEG LA website and imported into a file called MPEG Pool List, which was then used to extract a subset of complete patent records from the NBER patent file. Due to the nature of the available NBER data on between-patent citations, the calculations are limited to patents issued between the years of 1975 and 1999. In the case of the MPEG patent pool, four patents fall outside the upper bound of this date range. Calculations for the MPEG pool were made with the exclusion of these four patents. Sixty-one intra-pool citations were also extracted from the patent citation file for use in calculating the patent network density of the pool, Δ_p. Using a software package called Pajek, the MPEG pool and all of the intra-pool citations can be represented visually using a technique called force-directed graph placement (Fruchterman & Reingold 1991), as shown in Figure 3.

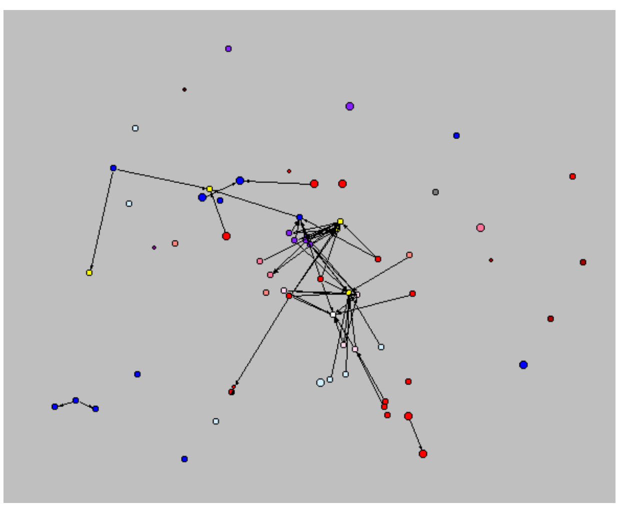

Figure 3. The MPEG Pool and Intra-Network Citations.

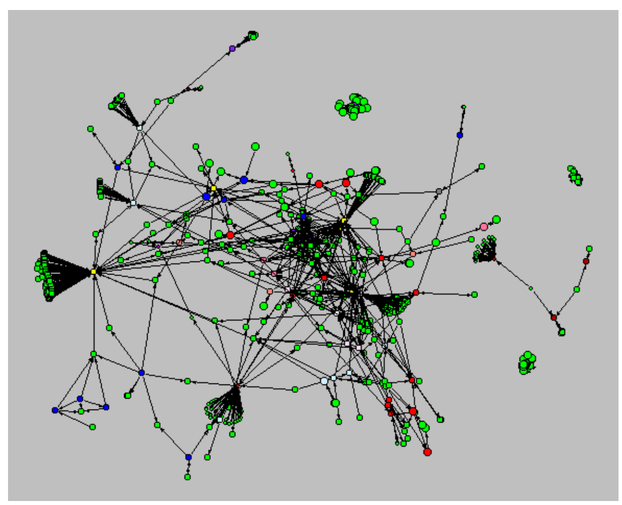

Figure 4. The MPEG Near-Universe and Intra-Network Citations.

a) Constructing the Complete Universe

Having downloaded the complete patent records for the MPEG pool, the next task was to construct a complete universe of citations based on the patent class memberships of the various

MPEG patents. A total of nine 3-digit patent classes were represented in the MPEG pool,[12] and a list of patent records was extracted that corresponded to the time period defined as the range between the oldest MPEG patent and the youngest MPEG patent. This extraction resulted in an MPEG Class Universe of 72,761 patents. Using that set of patents as a set of network nodes, 501,346 intra-network citations were extracted from the NBER citation file for use in calculating Δ_p for the complete universe. For comparison purposes, similar "complete" universe examples were also generated using NBER constructed measures for technology subcategory[13] and category.[14] A final universe was also generated with only a time restriction.

b) Constructing the Relevant Near Universe

Even when limited to certain patent classes over a certain period of time, a complete patent universe can be quite large, as is evident from the size of the MPEG universe. For a given patent pool, a subset of the surrounding universe may provide sufficient differentiation to identify that the pool is coincident with a patent thicket. Construction of the nearby universe involves a technique called a "snowball sample." Starting with the list of patents, in this case the MPEG pool patents, citations are extracted from the NBER citation file for patents that either cite to a pool member or are cited by a pool member. As with the complete universe, in addition to a near universe based on the 3-digit patent classes corresponding to MPEG technology, near universes were also generated for technology subcategory and category. Patent network density values Δ_p were then calculated for each near universe. Using the same force-directed placement graphing technique as before, it is also possible to represent the MPEG pool visually within its surrounding near universe with all of the intra-network citations, as shown in Figure 2 above.

[12] The MPEG patents came from patent classes 341,348, 358, 369, 370, 375, 382, 386, and 714.

[13] Subcategories 21, 22, 24, and 49.

[14] Categories 2 and 4.

c) Comparison of Network Densities

Table 5 presents the various Δ_p calculations for MPEG and t-test results for average density comparisons.[15]

Obs	Patents	Citations	Average Density Δp	Comparison of Δp against Δp for			
				Obs 1	Obs 2	Obs 3	Obs 4
1	65	61	0.029327				
2	265	573	0.016381	0.0085			
3	935	3521	0.008064	< .0001	< .0001		
4	958	3598	0.007849	< .0001	< .0001	0.7177	
5	72761	501346	0.000189	< .0001	< .0001	< .0001	< .0001
6	335781	2203322	0.000039	< .0001	< .0001	< .0001	< .0001
7	790078	4519205	0.000015	< .0001	< .0001	< .0001	< .0001
8	1765311	10566170	0.000007	< .0001	< .0001	< .0001	< .0001

Table 5. Density Comparisons for MPEG.

- Observation 1: MPEG Intra-pool density. Only in-pool citations
- Observation 2: MPEG patent density within near universe (by *n* classes 341, 358, 369, 370, 375, 382, 386, and 714).
- Observation 3: MPEG patent density within near universe (by subcategories 21, 22, 24, and 49).
- Observation 4: MPEG patent density within near universe (by categories 2 and 4).

As is evident from Table 6, the density of the MPEG patent pool is statistically differentiable from any of the three near universe densities as well as any of the four complete universe densities.

d) Examination of Shadow Pools

Two MPEG shadow pools were also created. The first shadow pool was constructed by iterating through each MPEG patent in the pool and selecting the next sequential patent from the MPEG Class Universe. A second shadow pool was created by selecting the patent immediately preceding each MPEG pool patent.[16] Intra-pool citations were then extracted from the NBER citation database for each of these shadow pools to facilitate Δ_p calculations. When the densities

[15] For the t-tests, variance is not assumed to be equal.

[16] Each of these shadow pools has one less patent than the MPEG pool, as shifting forward or backward by one patent results in the truncation of either the earliest or latest patent, since the time period for this analysis is defined as beginning with the oldest MPEG patent and ending with the youngest MPEG patent.

from the two shadow pools were compared against the complete universe, however, they were not statistically different from the complete *n class* universe (Observation 5).

2. Analyzing the PRK Pool

The list of PRK patents was developed from the FTC complaint, and a set of patent records was extracted from the NBER patent database. Using the same methodology as with the MPEG pool, patent networks were created for the PRK pool, the universe near the PRK pool, and a complete universe based on the two 3-digit patent classes[17] that covered the PRK pool, one each for the NBER- constructed variables of technology subcategory[18] and category,[19] and one universe with no technological constraint. Table 6 presents the various Δ_p calculations for PRK:

Obs	Patents	Citations	Average Density Δp	Comparison of Δp against Δp for			
				Obs 1	Obs 2	Obs 3	Obs 4
1	25	61	0.20333				
2	197	1324	0.06858	0.0069			
3	239	1548	0.054428	0.0032	0.0837		
4	259	1602	0.047948	0.0023	0.0085	0.3375	
5	17138	188601	0.0012843	0.0002	< .0001	< .0001	< .0001
6	87205	902621	0.0002374	0.0002	< .0001	< .0001	< .0001
7	204199	1452254	0.0000697	0.0002	< .0001	< .0001	< .0001
8	1765311	10566170	0.0000068	0.0002	< .0001	< .0001	< .0001

Table 6. Density Comparisons for PRK.

- Observation 1: PRK Intra-pool density. Only in-pool citations
- Observation 2: PRK patent density within near universe (by *n* classes 351 and 606).
- Observation 3: PRK patent density within near universe (by subcategories 32 and 39).
- Observation 4: PRK patent density within near universe (by category 3).

The numerical analysis of the PRK pool leads to the conclusion that the PRK pool is coincident with a thicket, as does a visual examination of renderings of the PRK pool network and its nearby universe, as shown in Figures 5 and 6.

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[17] Patent classes 351 and 606.

[18] Subcategories 32 and 39

[19] Category 3

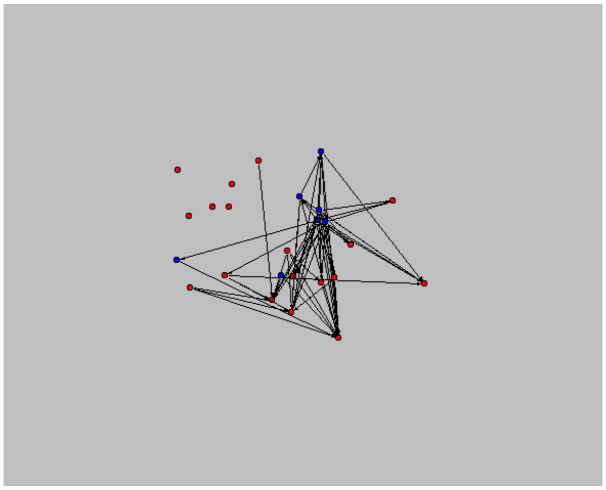

Figure 5. Laser Eye Surgery. PRK Pool Intra-Network Citations.

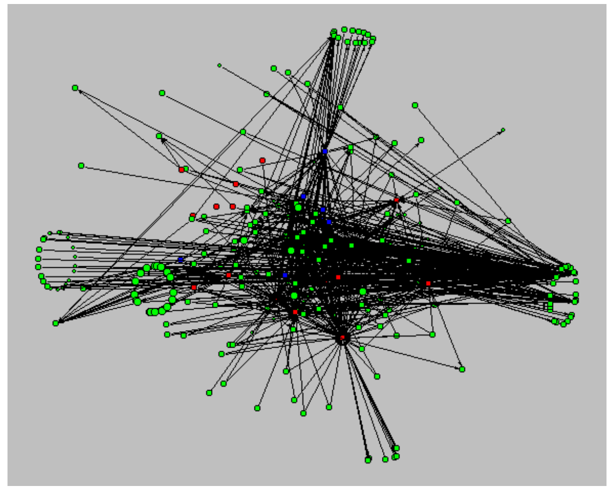

Figure 6. Laser Eye Surgery. The PRK Pool's Near-Universe.

3. Potential for False Positives

One issue that should be addressed involves whether in certain cases many cross-citations exist in the absence of a surrounding thicket. Cross-citation patterns of varying density will exist within patent space, and higher density areas may or may not be thickets. Shapiro's definition of a patent thicket is driven by an applied market interpretation concerned with the implications of patent groups for antitrust scrutiny. The proposed methodology assumes that market forces lead to the formation of a patent pool, and that the citation patterns present in the pool would also be present in a patent thicket even in the absence of a pool. Thus, while there may be areas of higher patent network density that have not been aggregated into a pool, Shapiro's definition suggests, and the analysis thus far confirms, that patent pools are coincident with underlying patent thickets.

C. Analytic Conclusions

Although there are identifiable differences between the pools, an objective analysis, both of their contract provisions (Clarkson 2004b) and the nodal interrelationships between their respective patents, indicates that they are not necessarily that different. As mentioned previously, the primary difference appears to be the existence of a technology standard in the case of the MPEG pool versus the absence of such a standard in the case of the PRK pool. The availability of the standard allowed MPEG to point to a third party determination of the existence of a patent thicket, while the patent thicket underlying the PRK pool was only proven after years of prohibitively expensive litigation.

This result is not necessarily surprising given the relative paucity of examinations of patent interrelationships in the legal case histories (Clarkson 2004a). In those cases the overriding factor in most of the decisions was the presence or absence of restrictive licensing terms. Gilbert found a similar contractual focus in the cases he reviewed (2002), even though he makes a strong argument that the competitive relationships between the patents should be the most important factor in assessing the pro-competitiveness of a given pool.

From a patent density standpoint, the analysis in Part V.B. demonstrates that both the MPEG pool and the PRK pool were coincident with patent thickets. If the antitrust regime were to adopt an objective methodology for identifying the existence of underlying patent thickets, patent pools formed outside of the standards-based context might have a chance at survival.

D. Likelihood of an Alternate Outcome

Given the proposition that the PRK pool was ultimately destroyed because the pool was unable to demonstrate the existence of an underlying patent thicket, it seems appropriate to pose the same question that was asked about the historical litigation: Would the availability of an objective methodology for determining the existence of a patent thicket have made any difference for either the MPEG or PRK pools? In the case of MPEG, such a methodology would have only confirmed what the DOJ was already willing to accept, that the process for determining essentialness also defined the boundaries of the underlying patent thicket. In the case of the PRK pool, however, the answer is a resounding “YES!” The admissions of former and current FTC litigators involved in the case clearly demonstrates the point that, absent the allegation of fraud regarding one particular patent, the patent pool would probably not have been attacked. With an objective confirmation of the existence of an underlying patent thicket, however, the FTC could have left the pool intact if it legitimately believed that the patent in question was invalid and could have instead focused on challenging the patent. If that patent were ultimately found to be invalid, then it could have been pulled from the pool, a remedy that seemed to satisfy the DOJ in its review of the MPEG pool. [20]

While the FTC’s actions to force the PRK pool members to enter into a cross-license did not allow the PRK patent thicket to reform relative to the initial pool members, it did nothing to solve the patent thicket problem relative to new entrants. In fact, the relative litigation peace that the patent pool facilitated was shattered when it dissolved. At least six infringement suits were subsequently initiated because of conflicting claims about both PRK and a new, emerging procedure called LASIK,[21] which shared some of the core technological requirements potentially covered by the PRK patent pool (Clarkson 2004a). One pool member was so damaged by the litigation that it was quickly acquired by a larger healthcare conglomerate, and any potential new entrants faced the prospects of negotiating licenses with multiple potential competitors before they could break into the market (Clarkson 2004a).

[20] Letter from Joel Klein, June 26, 1997, footnote 40. (“The Department presumes from the information you have provided us that the Portfolio patents are valid. Should this prove not to be so, the Department's analysis and enforcement intentions would likely be very different. As noted above, the Agreement Among Licensors provides for the deletion from the Portfolio of licenses held invalid or unenforceable.”)

[21] LASIK stands for Laser-Assisted *In situ* Keratomileusis.

The availability of an objective methodology for determining the existence of an underlying patent thicket could have fundamentally altered the course of the litigation, left the pool in place, and produced a better outcome in terms of social welfare.

VI. Discussion

A. Contributions

In proposing an objective methodology for the determination of the existence of patent thickets, this paper makes a number of contributions to both the intellectual property and antitrust regimes. It also adds a third component to the antitrust analysis of patent pools beyond contract structure and market dynamics by facilitating an inquiry into the existence of an underlying patent thicket. The methodology also provides a potential mechanism for allocation of antitrust enforcement resources. Finally, the methodology provides an alternative method of thicket identification in instances where a technology standard is not available.

The antitrust enforcement regime often uses thresholds to allocate enforcement resources. A prime example is the use of the Herfindahl-Hirschman Index ("HHI") of market concentration. The incorporation of the HHI into the antitrust regime shifted the analysis of horizontal mergers from one of intent, highly subjective and often difficult to assess conclusively, to one of objectively determinable market characteristics (Viscusi *et al.* 2000). "Mergers resulting in unconcentrated markets[, HHI index below 1000,] are unlikely to have adverse competitive effects and ordinarily require no further analysis" (USDOJ 1992, § 1.51(b)) even if the merging parties have evil intent. Conversely, mergers involving angelic participants with no ill intent whatsoever will be examined thoroughly if they substantially alter the HHI in a highly concentrated market (i.e. more than a 50 point change in a market with an HHI greater than 1800).[22]

The thicket identification methodology would allow antitrust enforcement officials to narrow the set of issues that needs to be examined for a patent pool if the existence of an underlying patent thicket can be demonstrated. If combinations of patents are not coincident with an underlying patent thicket, the combination is unlikely to be welfare-enhancing and thus deserves heightened scrutiny and detailed examination of contract structure.

B. Limitations

As with the aphorism "All poodles are dogs, but not all dogs are poodles," all patent pool members may fall within a patent thicket, but not all patent thicket members should be allowed in pools from a policy standpoint. Whereas density seems to be a sufficient measure for thicket identification, a different type of analysis is needed in order to identify whether or not a patent pool is a pro-competitive solution for a given set of patents in a thicket. In order to assess the pro-competitive benefits of a given patent pool, it is necessary to peer inside the patent thicket and determine how the individual patents relate to each other. This determination requires an assessment of the BCIS relationships, which the proposed methodology for patent thicket identification cannot address.

While the proposed methodology facilitates a determination of whether or not a given pool satisfies the necessary condition that an underlying patent thicket exists, that condition is

[22] There is, however, a fundamental difference between the proposed methodology for patent thicket identification and the HHI, at least for the moment. The threshold levels of the HHI are applied across industries. Thus a post-merger HHI of 1900 indicates a highly concentrated market regardless of industry. This paper does not assume that the density of one thicket can be compared to the density of another. What is suggested is that given an area of patent space, the density of patent thickets, like the relative brightness of stars in the night sky, makes them differentiable from the background patent space. Further research may, however, ultimately lead to the development of threshold levels for the evaluation of patent thicket density, which would facilitate density comparisons between industries.

necessary but insufficient, as mentioned previously. Since potential anticompetitive harms from patent pooling still exist (Priest 1977), the antitrust enforcement regime should not automatically approve any pool submitted for review, particularly in light of the anti-competitive history of patent pooling in the first half of the 20th century (Carlson 1999; Gilbert 2002), even if the proposed pool can be demonstrated to be coincident with an underlying patent thicket. The elimination of substitutes is also a necessary but insufficient condition for a pool to be pro-competitive.

Although the proposed methodology does not attempt to assess the BCIS relationships within a patent thicket, this paper is the first attempt to objectively demonstrate the existence of patent thickets. Without first verifying the existence of a given thicket, any attempts to objectively segment a thicket into BCIS categories would likely be futile. Answering the question of thicket existence is thus a first step toward empirical usefulness of the theoretical BCIS framework. The proposed methodology provides a foundation for further exploration of the nature of patent thickets and the development of policies to facilitate the formation of pro-competitive patent pools to solve the problem posed by patent thickets.

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